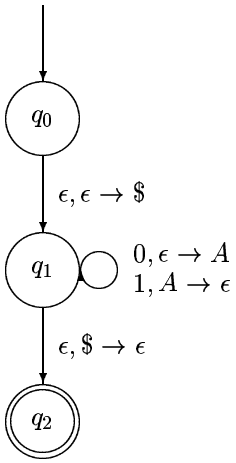
Problem Construct the CFL accepting the language described by the PDA



over the input alphabet $\Sigma = \{0, 1\}$ and stack alphabet $\Gamma = \{A, \$\}$, according to the construction given in class and in **Lemma 2.15** of Sipser. Finally, prune the rules in the resulting CFL to produce a CFL for the same language, but with a small number of rules (*Hint*: You should end up with four rules).

Answer Step 1:

$$\begin{array}{rcl} (q_1, \$) & \in & \delta(q_0, \epsilon, \epsilon) \\ (q_2, \epsilon) & \in & \delta(q_1, \epsilon, \$) \end{array} \quad \Rightarrow \quad A_{02} \to A_{11}$$

$$\begin{array}{rcl} (q_1, A) & \in & \delta(q_1, 0, \epsilon) \\ (q_1, \epsilon) & \in & \delta(q_1, 1, A) \end{array} \quad \Rightarrow \quad A_{11} \to 0A_{11}1$$

Step 2:

$$\begin{array}{rcl}
A_{00} & \to & A_{00}A_{00} \mid A_{01}A_{10} \mid A_{02}A_{20} \\
A_{11} & \to & A_{10}A_{01} \mid A_{11}A_{11} \mid A_{12}A_{21} \\
A_{22} & \to & A_{20}A_{02} \mid A_{21}A_{12} \mid A_{22}A_{22} \\
A_{01} & \to & A_{00}A_{01} \mid A_{01}A_{11} \mid A_{02}A_{21} \\
A_{10} & \to & A_{10}A_{00} \mid A_{11}A_{10} \mid A_{12}A_{20} \\
A_{02} & \to & A_{00}A_{02} \mid A_{01}A_{12} \mid A_{02}A_{22} \\
A_{20} & \to & A_{20}A_{00} \mid A_{21}A_{10} \mid A_{22}A_{20} \\
A_{12} & \to & A_{10}A_{02} \mid A_{11}A_{12} \mid A_{12}A_{22} \\
A_{21} & \to & A_{20}A_{01} \mid A_{21}A_{11} \mid A_{22}A_{21}
\end{array}$$

Step 3:

$$\begin{array}{rcl} A_{00} & \to & \epsilon \\ A_{11} & \to & \epsilon \\ A_{22} & \to & \epsilon \end{array}$$

Now combine everything and prune:

$$\begin{array}{rcl} A_{02} & \to & A_{11} \\ A_{11} & \to & 0A_{11}1 \mid A_{11}A_{11} \mid \epsilon \end{array}$$

A_{02} is the start symbol.